Some Observations on Modeling and Simulation of Spacecraft Formations

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Abstract

In this paper, an investigation is made of the reconfiguration dynamics of spacecraft formations. We introduce multibody dynamics concepts in the formulation, and discuss the characteristic dynamic modes involving multiple scales. The *virtual structure* concept is introduced and will be useful in determining the stability conditions of a formation when a feedback controller is closed between the formation members. The stochastic dynamics of formations is also proposed as a means to analyze swarms of spacecraft in orbit.

1 Introduction

The revolutionary vision of formation flying spacecraft is in contrast to the traditional approach of deployment of large and expensive multiple payloads platforms. Several applications of formation flying have been proposed by NASA and other agencies [6], [7]. Formation flying spacecraft must conform to extremely stringent control and knowledge requirements. Requirements of such precision have never existed before. For example, the control system for space interferometry must provide precision station-keeping for both coarse requirements (relative position control of any two spacecraft to less than 1 cm, and relative attitude control of 1 arcmin over a large range of separation from a few meters to tens of kilometers) and fine requirements (nanometer relative position control, and .01 milliarcsec relative attitude control). Conformity to such precise performance metrics presents new challenges, not only in the areas of guidance, estimation, and control, but also in the areas of dynamic modeling of the formation flying spacecraft and its environment. It is crucial to be able to better understand and model physical effects which would have been deemed unimportant or secondary for less precise spacecraft control applications. Several publications have already appear that address some of the complexities of formation flying [3], [4], [5]. In this paper, we derive the equations of motion for a generic model of a formation of N orbiting bodies. The resulting model is used in simulation studies for a formation control research program at JPL. First, a general nonlinear model is derived. This model contains sufficient detail for control design applications. A linearized model is also derived, which is used to infer the structure of the system and to obtain insight into the dynamics, stability, and control of a formation. The observations reported in this paper clearly identify the differences between the dynamics of a conventional spacecraft and the dynamics of a formation of different spacecraft which operate synchronously to achieve a common objective. We also propose a novel algorithm to track a reconfiguration profile, when the equations are written as if the formation were a multibody problem. This allows an explicit expression for the control law. A formation may be looked at as a constrained multibody system. In this particular view, two types of constraints are introduced between the bodies of the formation. This formulation immediately leads to posing the problem of how to explicitly compute the actuator forces and torques which allow for complete plant inversion, once the trajectories of the bodies of the formation are specified. These trajectories may be defined following criteria such as fuel optimality, collision avoidance, and generic mission constraints, thereby making the proposed control effort computation scheme quite general for the determination of fuel consumption estimates. Finally, we address the stochastic dynamics of ensembles of spacecraft as a tool to analyze larger formations, or swarms of spacecraft performing cooperatively. The development of techniques to ensure stability, performance, and efficiency throughout various stages of formation flying mission has been an active area of research in recent years. This body of work has focused on formation modeling, attitude coordination, formation geometry, autonomous formation reconfiguration, time constraints, fuel efficiency, maneuver optimality and collision avoidance. The purpose of this paper is to provide an overview of the fundamental issues in the areas of modeling of formation flying spacecraft with references to the most recent theoretical and experimental research developments in this area conducted by the author.

2 Modeling and Simulation of Formations

The development of models and the design of simulation techniques for formation flying spacecraft poses significant challenges compared to those of conventional spacecraft. Since a formation can be defined as a distributed spacecraft composed of physically disconnected vehicles, this fact only leads to an uncommon way to analytically represent its dynamics. The derivation of reduced order dynamical models for control, and the need to conveniently represent external perturbations and modeling uncertainties entering the model, also pose challenging problems. From a dynamical standpoint, a formation of spacecraft is characterized by both a wide dynamic range (from less than 1 Hz for individual spacecraft dynamics to KHz in the operation of the entire formation), and by spatial scales ranging from sub-micron to kilometers. To date, techniques to model such systems do not yet exist.

The new capabilities enabled by a precision formation flying spacecraft will require significantly higher fidelity modeling and simulation of the flight system as well as computational architectures to help develop, test, and validate distributed spacecraft missions. Never before has modeling fidelity been more strongly driven than precision formation flying requirements. The formation can be thought of a virtual truss in which the stiffness and dissipation levels of the connecting links are dictated by the control action on the relative sensing and actuation between two or more neighboring spacecraft. The dynamic model of this virtual truss suffers from undesired deformation modes caused by sensor noise, actuator non-linearity, dynamic uncertainties, and environmental disturbances. Some of these perturbations are stochastic in nature, others are well predicted by deterministic models. In light of the unprecedented, extremely fine performance requirements, a comprehensive modeling of all uncertainties becomes far more important for a formation flying spacecraft than for a conventional spacecraft. Specifically, in a low Earth orbit, orbital dynamics and environmental disturbances introduce additional strong non-uniform, nonlinear dynamic perturbations to each spacecraft in the formation. The formation control model can then be used to develop control laws and validate performance requirements. The control model for stationkeeping dynamics includes linearized models of the open-loop dynamics of each spacecraft, controller and estimator induced state coupling of the formation geometry, sensor/actuator dynamics, and sensor/actuator location. The formation models for control design are generally different from the formation dynamic models. The latter captures nonlinear models of the open-loop dynamics of each spacecraft. This includes representative models of sensor/actuator dynamics and locations and nonlinear models of the orbital dynamic effects of Earth and Sun. The dynamic model also incorporates nonlinear models of the environmental perturbations induced by Earth magnetic field, Earth radiation pressure, solar pressure, harmonics of gravitational potential, gravity gradient disturbances, thermal effects of solar illumination and Earth albedo. Formation reconfigurations, instead, require fully nonlinear models. In this case, an efficient description is needed of the absolute and relative translational and rotational dynamics of the entire formation. The formation geometry can be expressed in terms of the states of a spacecraft in the formation and the states of the remaining spacecraft relative to the designated reference spacecraft. This naturally introduces an effective coupling between all spacecraft states that must be maintained throughout the formation. Different scales of motion occur simultaneously in a formation: translations and rotations of the formation as a whole (macro-dynamics), relative rotation and translation of one formation member with respect to another (relative dynamics), and formation member flexibility (micro-dynamics). A challenge is to incorporate these modes of motion into a representative reduced order model. The formation models derived in this paper are models used to develop control laws and validate performance requirements.

2.1 Multiple dynamic scales and novel formation modeling techniques.

A new scheme for representing the dynamics of the formation is presented that allows to analyze different classes of problems involving general orbiting formations. As a consequence, a rigorous framework will be available enabling the analysis of general N-body formations, fleets, constellations, or collections of formations, undergoing synchronous or asynchronous motion, possibly located in different orbits. We envision formations of distinct types: from a small number of moderate-sized spacecraft carrying deployable reflectors to hundreds or more microspacecraft equipped with autonomous or semi-autonomous attitude, navigation, and control system on board, designed to map extensive domains of the geosphere, form communication networks, or act as distributed space warning and surveillance systems. These systems are capable of responding and altering their configuration in an autonomous manner to external stimuli such as, for instance, an increased solar activity or the requirement of more extensive Earth coverage upon request from ground. This necessitates extremely flexible reconfiguration capabilities, as well as the ability to change the topology of the graph representing the visibility of one spacecraft with respect to another. First, we look at a formation as an ensemble of objects which can be described using tools from graph-theory. Some structural properties emerge which are typical of formations, and which do not have a counterpart in the modeling of conventional spacecraft. In particular, starting from the assumption that a formation is composed of objects continuously controlling their relative position and attitude, one can define an equivalent structure which overlays the formation at each instant of time. We call this template the virtual truss. This terminology is mostly for graphical purposes since, in general, a truss responds only by relative motions along the connecting line between two members. A more descriptive term would be *virtual body* (or virtual truss), since relative position and orientation adjustments are also possible. In order to identify the possibility of relative motions existing inside a formation, one needs to look at different scales of observation. This leads to the concept of *formation modes.* For the virtual truss, we show that modes of the formation exist in the sense that an associated eigenvalue problem can be obtained from a solution of a self-adjoint boundary value problem problem related to the formation kinetics. For a formation of N rigid objects, each with 6 degrees of freedom, the equivalent virtual truss will show a pattern of standing waves representing $6 \times N$ modes of deformation. These modes are analogous to the modes of a vibrating structure, but they originate from a completely different type of source. We also attempt a unification of the deformation and dynamic modes of a formation, and this leads us to use the kinematics of microcontinuum field theory to describe the motion of individual units of a much larger formation. Each individual unit is endowed with a position vector, a rotation tensor, and a deformation gradient tensor, in the spirit of micromorphic kinematics. This means that each individual unit, henceforth called a *particle*, is capable of changing its configuration in response to stimuli originated either from the exterior of the formation or within the formation itself. The formation is therefore treated as a continuum at the macroscopic level, with added extra structure at the microcontinuum, or particle, level. Kinematically, it corresponds an energetically conjugate description of the kinetics based on measures of internal reconfiguration stresses. A set of balance laws for the formation can then be derived, assuming invariance of the formation energy functional under translations and rotations. These balance laws include the conservation of the formation mass, the balance of formation linear momentum, the balance of macroscopic formation angular momentum and of particle angular momentum, the formation entropy inequality, and the boundary conditions at the boundary of the formation. The description of the internal constitution of the formation, i.e. the constitutive relation between internal reconfiguration kinematic variables (strains) and internal reconfiguration momenta, completes the mechanical description of the formation. The internal reconfiguration momenta represent the generalized inertia and the generalized stresses that the particle experiences when a reconfiguration is taking place. The constitutive functional includes memory dependent terms and nonlocality in the formation response. This effect must be included due to the fact that the behavior of the formation can be influenced both at the system level and at the particle level. Therefore, two time scales enter the picture, as well as two spatial scales. The particle dynamics begins to emerge when $\frac{\lambda}{\tau} \approx 1$, where λ is the time (or space) scale of the stimuli internal or external to the formation, whereas τ is a time (or space) scale representative of the formation itself. Conversely, when $\frac{\lambda}{\tau} \ll 1$, the particle behavior is predominant, and when $\frac{\lambda}{\tau} \gg 1$, the formation behavior as a whole is predominant. Spatial nonlocality occurs since one particle may respond to stimuli from another particle located far away from it in the formation. Further, it occurs at a global level, since each particle may respond to stimuli of the formation as an individual entity. This multilevel behavior is reflected in the nonlocal constitutive functional. Memory dependence, also known as time nonlocality, enters the constitutive functional through time dependence of the current instant from previous instants. In this paper, we will describe the kinematics and the measures of formation and particle configuration, the dynamics and the measures of particle and formation internal response, the generalized functional which describes the formation behavior as an individual entity, and how this functional is related with the individual particle response.

2.2 The reconfiguration of a formation as a multibody problem

The algorithm hereby proposed can track any conceivable profile of interspacecraft distance $L(t)$ and orientation $\varphi(t)$ to realize reconfigurations with collision avoidance. It is also extremely useful since it provides actuator force and torque profile vs. time and estimate fuel and power consumption during formation reconfigurations. The algorithm makes use of the Singular Value Decomposition (SVD) to project the equations of motion of the constrained formation into the tangent subspace of the motion, which eliminates reaction forces and torques between pairs of interacting bodies. The tracking control input is then derived explicitly, as a function of the formation topology and time varying profile for $L(t)$ and $\varphi(t)$. It requires sensitivity matrices of constraints as a function of the degrees of freedom of the problem. It is independent of the number of bodies, and can be quickly generalized to very large formations, fleets, and separated formations. Thanks to a closed form expression, sensitivity analyses of control authority as a function of geometry and kinematics is also possible. The equations of motion may be written as [1] $\mathbf{M}\ddot{\boldsymbol{\eta}} + \Phi_q^T \boldsymbol{\lambda} = \mathbf{Q}$, with $\Phi_q \dot{\boldsymbol{\eta}} = \boldsymbol{\nu}(\mathbf{t})$, and with $\Phi_q \ddot{\boldsymbol{\eta}} = \boldsymbol{\gamma}(\mathbf{t})$. By introducing a coordinate transformation P such that $\boldsymbol{\eta} = \mathbf{P}_1 \mathbf{z}_1 + \mathbf{P}_2 \mathbf{z}_2$, where $\mathbf{P}_{1[n\times m]} = \mathbf{orth}\left(\Phi_q^T\right)$ and $\mathbf{P}_{2[n\times(n-m)]} = \mathbf{null}\left(\Phi_q\right)$, ($\mathbf{orth}\,(\cdot)$ represents the orthogonal complement, and $\mathbf{null}\,(\cdot)$ is the nullspace) one obtains a projection of the dynamics of the constrained system in a direction tangent to the constraint manifold. This means that the projected system moves in the direction of the kinematically admissible displacements, and the effect of the constraints on the balance of forces vanishes. This transformation is equivalent to the one obtained via a singular value decomposition of the constraint jacobian. Therefore, we have a way to eliminate the reaction forces from the equations of motion. This elimination process is exact, however it requires some extra computation at each integration time since the algebraic operations required by the SVD may be time consuming. Although the SVD decomposition has to be performed at each time step, causing some computational overhead, the algorithm is exact to within the limits of the SVD decomposition. This is satisfactory for the applications at hand. More demanding applications, in which the constraint violation approaches machine numerical precision, must be treated in a different manner. This is the case in which control algorithms must be designed and tested on a model of a formation required to maintain alignment and relative range and bearing constraints to tolerance levels approaching the machine precision of the simulation environment.

3 Conclusion

In this paper, we investigate the simplest dynamics models of a formation of spacecraft in orbit. After deriving the general equations of motion, we develop the stability conditions for the equivalent virtual structure in terms of the gains of the equivalent proportional-derivative feedback controller closed between members of the formation. These stability conditions are the first step towards understanding how to control these formations in a more general sense. We also address the reconfiguration dynamics and control of spacecraft formations using multibody dynamics concepts, obtaining closed-form solutions for the tracking control law. By imposing the constraints on relative position and attitude between the bodies, the equations take a different form. The advantage of this form is that, by means of a suitable variable transformation, the constraint reactions can be eliminated exactly, and an expression for the input forces and torques required to track a given formation profile can thus be obtained in closed form. The characteristic dynamic modes of general formations involving multiple scales and the *virtual structure* concept, are identified as necessary ingredients of a more general theoretical model of the dynamics of spacecraft in formation. Finally, the stochastic dynamics of formations is also proposed as a means to analyze swarms of spacecraft in orbit, and this relies on extracting the characteristic dynamical properties of a representative volume element of the swarm.

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